

EFFECT OF FOUNDATION FLEXIBILITY AND IMPEDANCE CONTRAST ON THE RESPONSE OF GRAVITY DAMS

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ABSTRACT

The complexity of the interactions between the three structural systems of gravity dam viz. dam-reservoir-foundation rock, compel the designers to adopt various simplified methods for the dynamic analysis. While these methods are necessary for the ease of computation they offer, it is imperative that interaction between the components of the coupled system be modeled properly and the results of these simplified methods be interpreted in an appropriate manner. This paper deals with study of responses of the dam to variation in the major material properties such as Young's modulus and Poisson's ratio for the foundation rock over a range of expected values. Koyna dam deepest non-overflow monolith has been analyzed for its response to the recorded Koyna ground motion under various impedance contrast values and Poisson's ratios. The properties of the dam concrete are treated as constants at the actual experimental values. Foundation and the structure are analyzed under 2-D linear elastic conditions. The results show that the Soil-Structure-Interaction is greatly influenced by the impedance contrast between the dam and the foundation. Comparison of the dam responses obtained in this study with the published literature and the actual behavior of the dam during 1967 Koyna earthquake, validates the study conducted and emphasizes the importance of correct values of material properties for the dam and foundations.

KEYWORDS: Gravity Dam, Earthquake Ground Motion, Soil-Structure Interaction, Acceleration Time History, Impedance Ratio

INTRODUCTION

Soil-structure-interaction (SSI) effect is the change in the response of the structure due to the changes in the ground motion characteristics as compared to the scenario had the structure not been there. The dams are founded on natural materials of varying composition having irregular joints and planes of weakness forming a heterogeneous structural supporting medium (FEMA, 2005). The most influencing foundation parameters affecting the SSI and hence the response of gravity dam are Elastic modulus (E_f), Poisson's ratio (ν_f) and damping (ξ). The magnitudes of elastic modulus and Poisson's ratio are seen to depend upon the applied dynamic loading due to earthquake excitation. The magnification of the Elastic modulus of concrete under dynamic conditions is a known fact. Raphael (1975) studied a large number of test results and indicated that the dynamic modulus of elasticity increases by 33% in compression and 24% in tension. Neville (1972), Prescu et al. (1985) found that the magnification of elastic modulus under dynamic condition could be upto twice the static modulus. Experimental determination of dynamic modulus on the cores extracted from the body of Koyna dam showed the value of 44,950 MPa as against a static modulus of 31,000 MPa giving a magnification of 45% (Gaikwad et al.,

2003). These values assume vital importance where SSI effect is included in the formulation in the gravity dam analysis. Out of the two major approaches of gravity dam analysis, the substructure approach carried out in frequency domain (Fenves and Chopra, 1984; Gupta, 2010) does not explicitly model the foundation. The standard FEM procedure, on the other hand, can model and analyse non-homogeneous geometrical and material properties for the foundation as existing in practical reality. This procedure is required to be carried out in time domain (Leger and Boughoufalah, 1988). Different values of E_f and ν_f can be assigned to different locations in the foundation and analysis can be done in one go. The variation of properties of the foundation rock can occur in any random direction. Lin et al. (2004) considered the variation in horizontal and vertical direction. In addition, actually the foundation soil may vary along the layers dipping in the upstream direction as well. In the present analysis the variation of the values of impedance ratio and Poisson's ratio along the depth and width of foundation is not considered, but a uniform value is assigned for the entire foundation. The objective of this paper is to study the effect of the variation in the response of the structure due to variation in the foundation properties, represented by impedance and Poisson's ratios. The material properties of the dam concrete are kept constant throughout this study at the average level and the foundation modulus and its Poisson's ratio are varied over a wide range. Large number of cases analysed with different combinations of these parameters have shown the marked influence the impedance ratio defined by the ratio of moduli of foundation rock and dam concrete, as well as the Poisson's ratio have on the dam response. Therefore, it is of vital importance to use correct values in dynamic analysis of gravity dams.

GOVERNING PROPERTIES

Impedance Ratio

It is a well-known fact that the response of the dam is more dependent on the impedance contrast represented by the ratio E_f/E_d , rather than the absolute values of the moduli of elasticity of either dam or foundation. Soil structure interaction is prominent when the foundation is flexible i.e. when E_f/E_d assumes lower values. Lin et al. (1988, 2004) reported that there will be reduction in the stress response of the dam with the reduction in the ratio E_f/E_d due to increase in fundamental period and the damping ratio of the dam-foundation system. Lokke and Chopra (2013) gave the period lengthening ratio and the addition to damping for different values of E_f/E_d ratio. Sarkar et al. (2007) studied the peak values of responses for different foundation moduli and found that with the decrease in foundation modulus, displacements increase. Lower the E_f/E_d ratio, lower would be the stress response of the dam. In the present study, the values of 0.25, 1.0, 3.0 and 5.0 are considered for E_f/E_d . The values of 0.25 and below indicate very soft material in the foundation which is not expected to be encountered in gravity dam foundations. At the same time, the value beyond 5.0 indicates a completely rigid foundation wherein the soil structure interaction does not take place and such a behavior is not expected in practical reality.

Poisson's Ratio

The range for the Poisson's ratio for foundation rock and dam concrete is found to be within 0.10 to 0.30 (Akroyd, 1962; Higginson, 1970; Prescu et al., 1985). The Poisson's ratio of concrete increases with its age, attaining a maximum value at about one year (Raphael, 1975). Under dynamic loading the value slightly increases upto 0.24. The value may go upto 0.27 for wet concrete. For dynamic analysis, USBR (1976) specifies a value of 0.20 only. In the present study, Poisson's ratio value of 0.10, 0.20 and 0.40 have been considered for different cases for the foundation, keeping the Poisson's ratio of dam concrete (ν_d) constant at 0.20.

Damping of the System

The damping of the system takes place under several complex mechanisms which dissipate part of the earthquake excitation energy. The practice has been to consider a small fraction of critical damping in the analysis (Tan and Chopra, 1995). The damping ratio between 5 to 10% of critical damping is recommended for mass concrete (Oberty, 1968). Further, the damping ratio increases with the order of the natural vibration mode. Wieland (2003) suggested damping ratios of about 10% for the lowest modes of vibration and even higher values for the higher modes for linear-elastic analysis of dam-foundation-reservoir systems. In view of these, it is appropriate to consider a viscous damping of 5%, 7.5% and 10% in the first three modes of vibration, which corresponds to hysteretic damping ratio of 10%, 15% and 20% of critical. For higher modes, if considered in the analysis, even though the damping value increases, its effect on the response of the structure is not significant as the contribution of higher modes to the total response of the structure is very small. Therefore, most of the times, an average value of damping is considered for all the modes.

Seismic Input

The seismic input in the form acceleration time history of ground motion applied in the standard FEM procedure gets magnified as they travel up the foundation extent because of the reflection of the seismic waves from the rigid boundary on the sides of the finite foundation mass. The magnification is due to the ground excitations which are free field accelerations recorded at the surface and applied at the base of the foundation in dynamic analysis by the standard FEM procedure. If a large foundation is considered, the error of applying the earthquake ground motions at the base of the foundation instead of at the ground surface might be significant. To alleviate these effects, massless foundation model is used by many investigators (Clough, 1980; USACE, 2003), wherein only stiffness and damping in the foundation is considered eliminating the contribution of the inertial forces. However, in this case the effect of soil structure interaction and radiation damping are ignored (FERC, 2006). Ignoring of dissipation of energy due to radiation damping in massless formulation could have significant effect on the stresses in dam (Chopra & Wang, 2010). The massless foundation formulation would enhance the response of the dam which according to Tan and Chopra (1995) and Chopra (2008) is of the order of a factor of two to three.

Another method of avoiding the erroneous magnification of ground motion in foundation is to deconvolve the ground acceleration to the modeled depth. Deconvolution of the free field ground motion to the depth of foundation is carried out to make them compatible to the depth of foundation considered in the analysis. The process of deconvolution of the horizontal component of the earthquake ground motion is performed considering the foundation as layered half space in which the viscoelastic shear wave propagates vertically upwards (Idriss and Sun, 1992; Kramer, 1996; Bardet, 2000). One dimensional site response programs like SHAKE (Schnabel et al. 1972), WAVES (Hart, 1989) and EERA (Bardet, 2000) simulate the non-linear soil behavior by equivalent linear technique for deconvolving the ground motion. But the large size of the foundation considered in order to eliminate spurious reflections of seismic waves in the foundation would pose difficulties in properly deconvolving the ground motions. And also, in reality the foundation irregularities can hardly be represented by modeling the foundation as purely stratified half space varying only in vertical direction as is done in the deconvolution procedure. Thus, if the varying parameters of the foundation rock are to be considered in the analysis, the recorded free field ground motion are required to be applied to the foundation in which all the irregularities as existing in reality are modeled. In the present study, deconvolution of ground motion is not done since the objective of the study is to compare the effects of variation in the impedance ratio and the Poisson's ratio for a given ground motion time history and

corrected peak ground acceleration (PGA) values 0.49g and 0.24g, respectively.

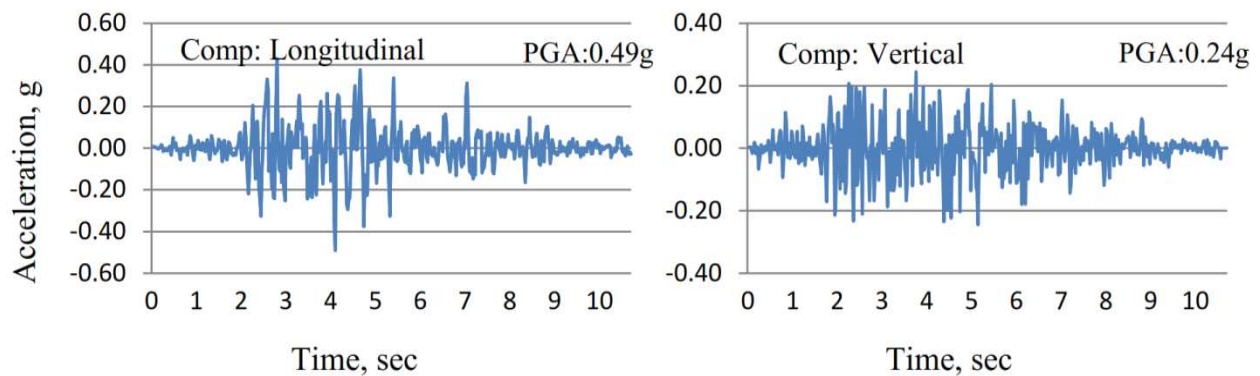


Figure 2: Horizontal and Vertical Components of Koyna Dam Accelerogram Used in the Present Study

Material Properties

The density of the dam concrete considered in the analysis is 2650 Kg/m^3 (Completion report of Koyna project, 1973). The density of foundation rock considered is 2800 Kg/m^3 . In the parametric study the values of modulus of elasticity and the Poisson's ratio for dam concrete are fixed at 44950 MPa and 0.2 respectively. The ratio E_f/E_d is varied as 0.25, 1.0, 3.0 and 5.0. The Poisson's ratio is varied as 0.1, 0.2 and 0.4 thus making 12 cases of analyses for the parametric study, as shown in Table 1.

Table 1: Cases and Data Considered for Parametric Study

Case No	Impedance Ratio (E_f/E_d)	Poisson's Ratio For Dam (ν_f)	Shear Wave Velocity (V_s)
1	0.25	0.1	1984
2	0.25	0.2	1984
3	0.25	0.4	1984
4	1.0	0.1	3968
5	1.0	0.2	3968
6	1.0	0.4	3968
7	3.0	0.1	6872
8	3.0	0.2	6872
9	3.0	0.4	6872
10	5.0	0.1	8872
11	5.0	0.2	8872
12	5.0	0.4	8872

RESULTS AND DISCUSSIONS

Parametric Study

The fundamental natural frequency of vibration for the dam along with the large foundation (351m x 206m) considered is a function of the stiffness of the system and the values computed in the present study are 1.07, 2.26, 6.90 and 7.14 Hz for the impedance ratios (E_f/E_d) of 0.25, 1.0, 3.0 and 5.0 respectively. Also, the natural frequency is found to reduce from 1.07 Hz to 1.01Hz with the increase in Poisson's ratio (ν_f) of the system from 0.1 to 0.2 and then it increases to 1.18 Hz for $\nu_f = 0.4$.

The peak values of the horizontal displacements of the upstream nodes of the dam section are plotted along the

height of the dam for different values of Poisson's ratio (ν_f) for a given value of E_f/E_d (Figure 3). Figure 4 is plot for displacement on upstream nodes for different ratio of E_f/E_d for a given value of ν_f . It is found that for very soft foundation represented by $E_f/E_d = 0.25$, the displacement response reduces for Poisson's ratio values from $\nu_f = 0.1$ to $\nu_f = 0.2$ and then it increases for $\nu_f = 0.4$. For $E_f/E_d = 1.0$, the displacement increases from $\nu_f = 0.1$ to $\nu_f = 0.2$. The displacement for $\nu_f = 0.4$ is in between these two values. For $E_f/E_d = 3.0$ and 5.0 , there is very little change in the displacement response of the dam as both these cases represent nearly a rigid foundation, wherein very little or no SSI effect is expected. In Figure 4, it is observed that for a given ν_f , the displacements reduce with increase in impedance ratio E_f/E_d , consistent with the behavior reported in the literature, (Lin et al. 2004; Sarkar et al. 2007 and Heirany & Ghaemian 2012). This observation is similar to the results of analysis in frequency domain as well.

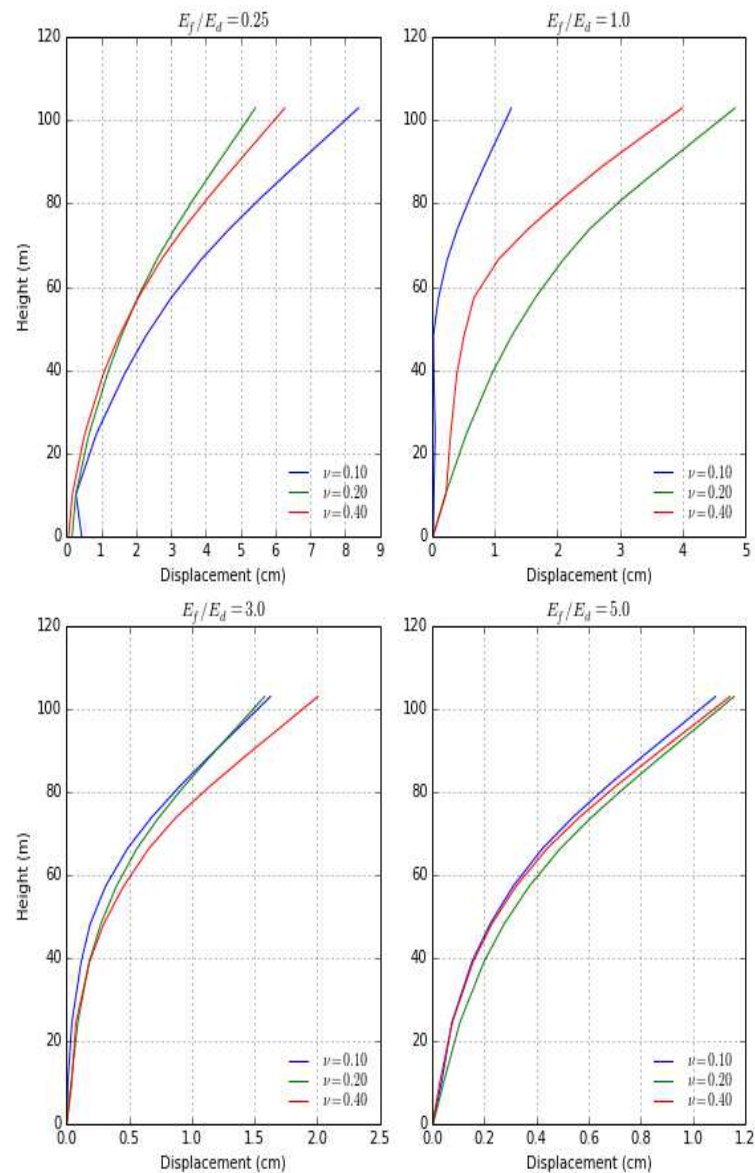


Figure 3: Peak Horizontal Displacements on the Upstream Nodes for Different Values of N_f for A Given Value of E_f/E_d Ratio

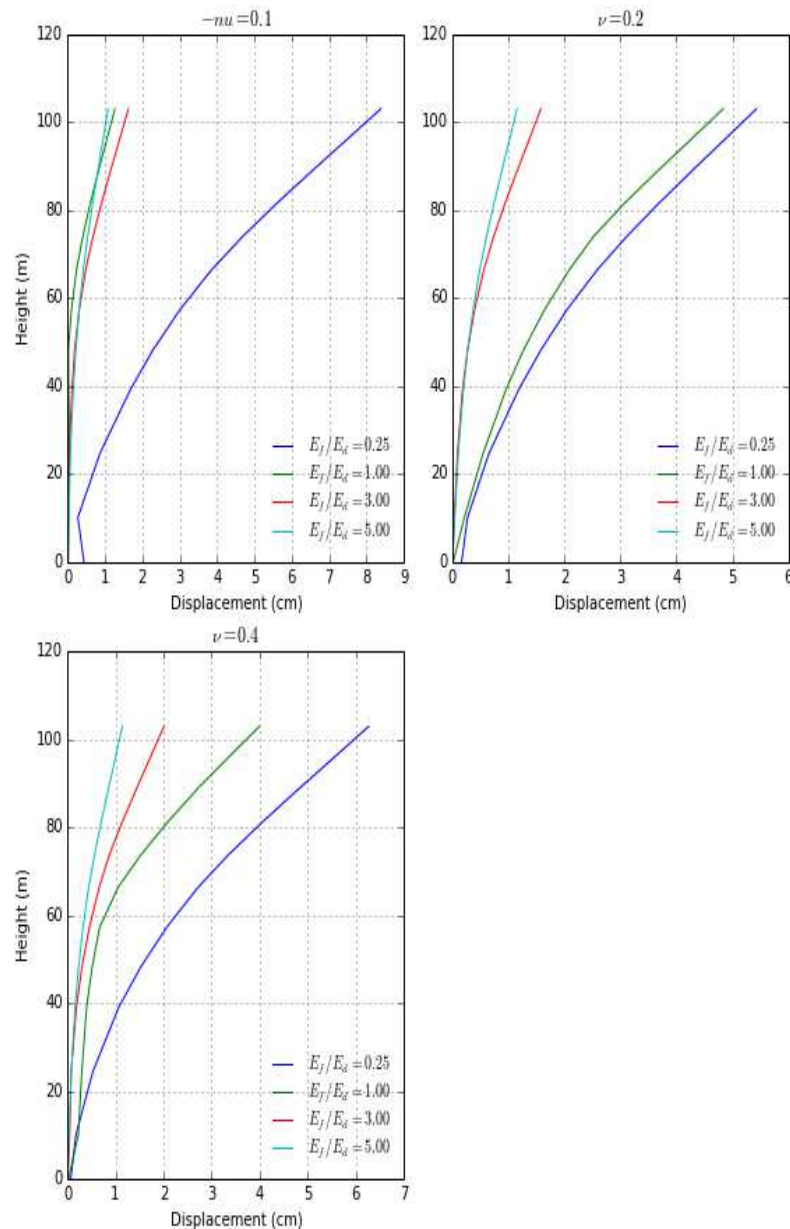


Figure 4: Peak Horizontal Displacements of the Upstream Nodes along the Height of the Dam for Different Values of E_f/E_d for a Given Value of ν_f

Figures 5 and 6 show the plots for the peak values of major principal stresses long the height of the dam. Even though no clear cut pattern is seen in the stress response as is seen in displacement response, the overall scenario is on the expected lines. The stresses at the top are nearly zero in all the cases and go on increasing generally towards the bottom, with a distinct zone of high stress in the middle, which is the point where the downstream slope suddenly changes. For $E_f/E_d = 3.0$ and 5.0 , there is a very high value of stress at the base indicating cracking of concrete at this location. The large impedance contrast results in considerable high stresses at the base. Material non-linearity has not been considered in the analysis and therefore, a crack is not introduced at the base in the formulation. This has resulted in returning high values of stress at the base and the stresses at the upper nodes are significantly smaller. Figure 6 gives the plot of peak values of major principal stresses for different values E_f/E_d for given values of ν_f .

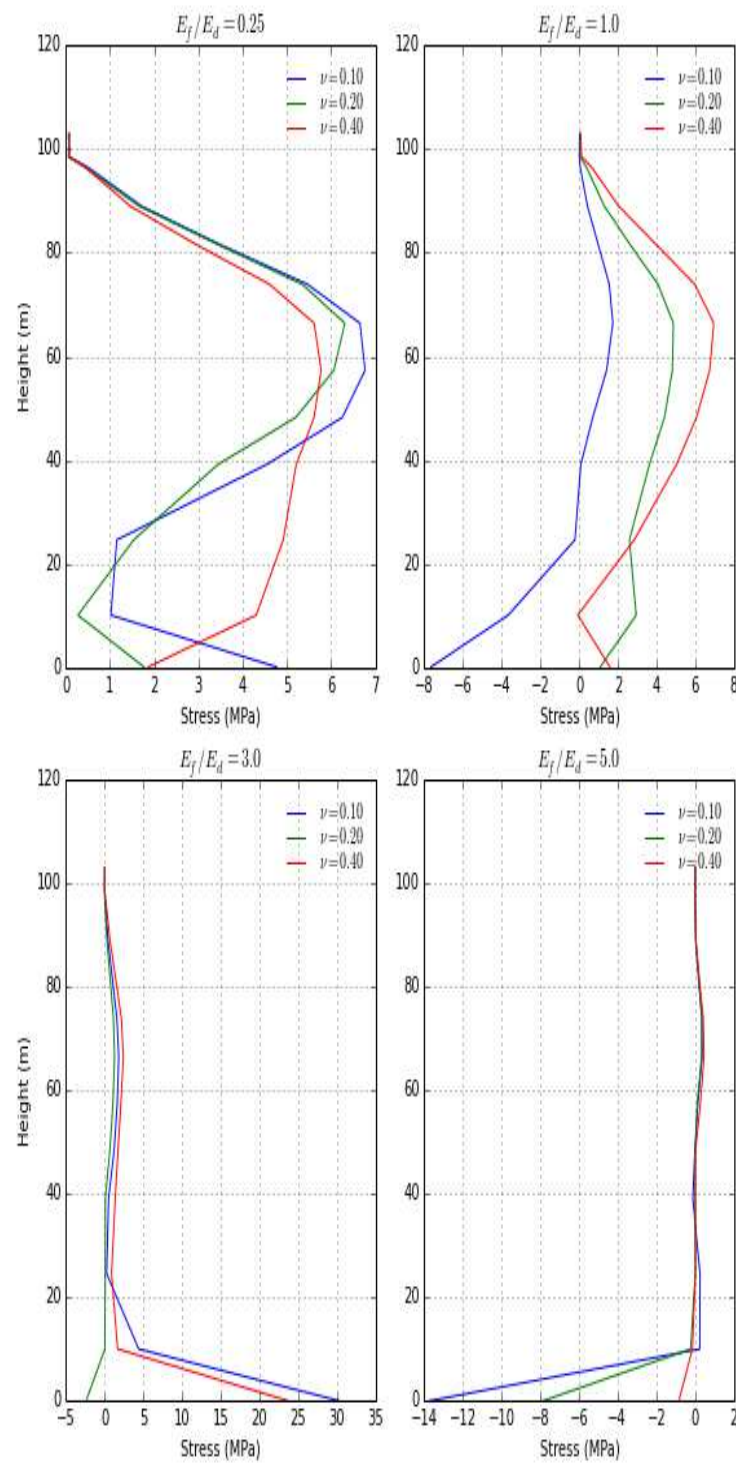


Figure 5: Major Principal Stresses on the Upstream Nodes for Different Values of N_f for a Given Value of E_f/E_d Ratio

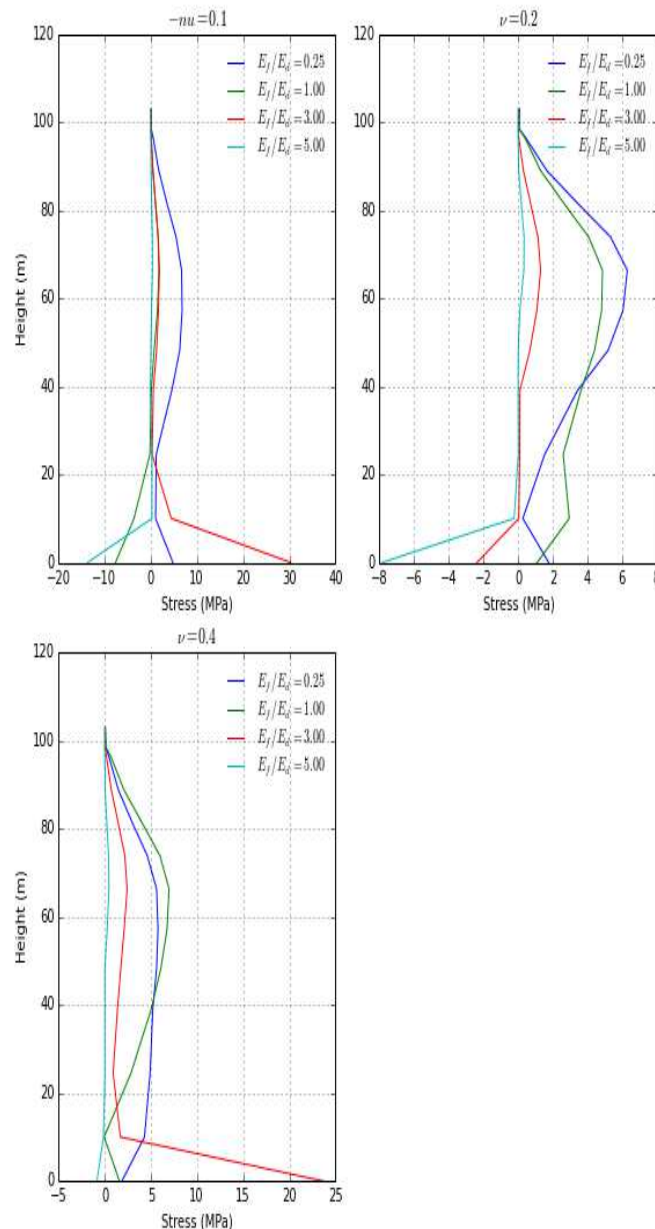


Figure 6: Major Principal Stresses on the Upstream Nodes for Different Values of E_f/E_d Ratio for a Given Value of N_f

It is also of interest to know the overall stress picture in the body of the dam. Therefore, contours of Major Principal stresses are plotted as shown in Figure 7 for a few cases of analyses with different combinations of impedance contrast and Poisson's ratio. These are in fact, maximum stresses that occur in each element during the entire duration of induced earthquake excitation. The area of cross section beyond the threshold stress value is used to find the safety evaluation of dam by computing demand-capacity ratio (USACE, 2007). For a given impedance ratio the area under higher stresses increases as Poisson's ratio increases from 0.1 to 0.2 and then again it reduces for $\nu_f=0.4$ as shown in Figures 7(a) and (b). This is similar to the observation for upstream stresses. Figure 7 (c) shows that the stress concentration near the base increases as the impedance ratio E_f/E_d increases for a given Poisson's ratio $\nu_f=0.20$. This again is similar to the observations in graphs for upstream stresses (Figures 5 and 6).

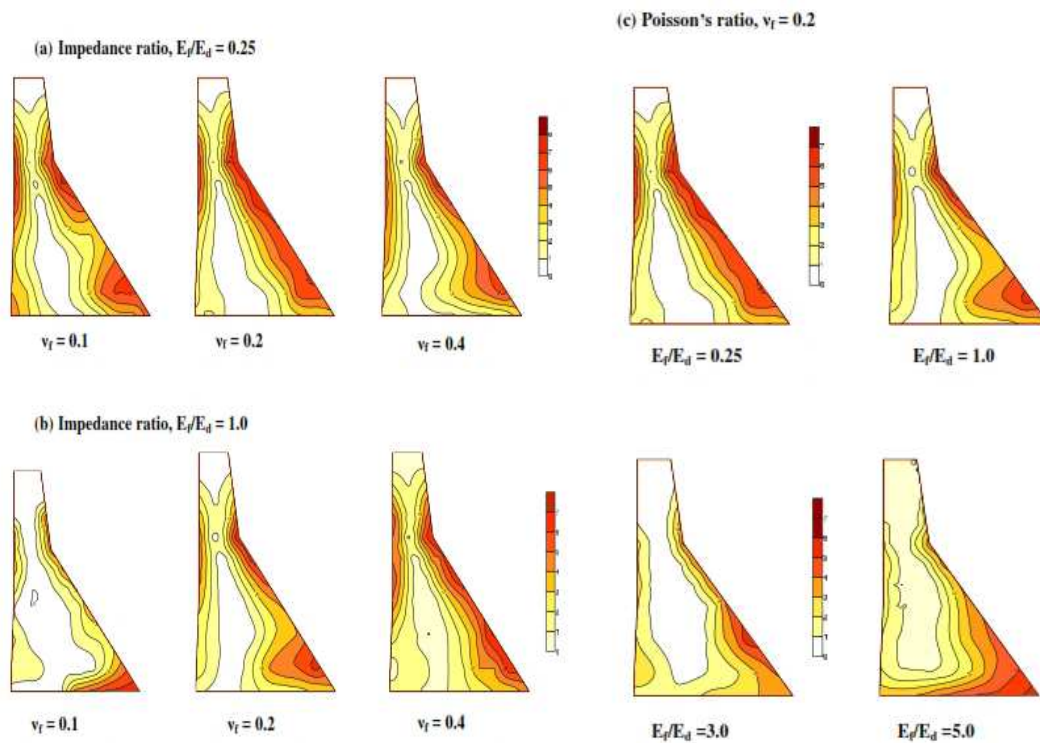


Figure 7: Major Principal Stress (MPa) Contours for Different Cases of Parametric Study

Response at the Neck of the Dam

In the present analysis, stress values of high magnitude have been found to occur at the neck of the dam on both upstream and downstream. This is due to the abnormal free board and top width of this old dam. The top width has been kept as 14% of the height (14.8 m) as was the practice in those days followed normally for small dams. This peculiarity has been identified by many investigators who have found the similar responses for stresses at the neck (e.g. Burman et al. 2010). This was the location where actual cracking was observed in Koyna dam during the 1967 earthquake (Chopra and Chakrabarti, 1973). In the present analysis, unlike the stresses on the heel of the dam, the stress at the neck of the dam increases with the decrease in the impedance of the foundation. This is due to the abnormal mass distribution along the height of the dam. There is a sudden change in the mass concentration of the dam section above the neck. The addition of mass at the top was done to incorporate raising of the dam for second stage of the project without changing the section from the foundation (completion report of Koyna project, 1973). The earlier concept of gravity dam design was to increase the weight of the dam without giving any regard to the location where mass was added. Table 2 shows the maximum principal stress on the upstream and downstream neck portion for different set of parameters considered in the analysis as mentioned in Table 1. Displacements at the neck for different cases are also presented in the Table 2.

Burman et al. (2010) reported maximum horizontal crest displacement of 7.04 cms for the linear elastic analysis of Koyna dam section for $E_f/E_d=0.55$. With the same dam section but with material non-linearity for the foundation rock idealized by Duncan-Chang model (1970) they observed a maximum displacement of 7.07 cms. Sarkar et al. (2007) obtained a horizontal crest displacement of 8.96cms for $E_f/E_d= 0.25$ with a slightly lower value of $E_d = 31,027$ MPa. In the present linear elastic analysis the horizontal crest displacement of 5 to 8cms have been observed for different values of E_f/E_d , with the modulus of dam concrete $E_d = 44,950$ MPa, and for values of $v_f = 0.1, 0.2$ and 0.4 . The values obtained in

the present study are for empty dam wherein hydrodynamic force is not included in the formulation. Further, the ground motion accelerations are not deconvolved in this analysis. Therefore, the responses are on the higher side for the empty dam. However, more than the absolute values of the responses, their dependence on the input parameters of impedance and Poisson's ratios are to be critically examined.

Table 2: Response of the Structure at the Neck for Different Parameters Considered in the Analysis

Case No	Impedance Ratio (E_f/E_d)	Poisson's Ratio for Dam (N_f)	Stress at the Neck (Mpa)		Displacement at Upstream Neck (Cm)	
			Upstream	Downstream	Horizontal	Vertical
1	0.25	0.1	6.65	12.00	3.82	3.24
2	0.25	0.2	6.31	12.95	2.56	2.09
3	0.25	0.4	5.62	9.51	2.68	0.20
4	1.0	0.1	1.75	4.61	0.23	0.45
5	1.0	0.2	4.87	8.71	2.07	0.39
6	1.0	0.4	6.97	11.97	1.02	0.51
7	3.0	0.1	1.84	4.13	0.47	0.17
8	3.0	0.2	1.30	3.29	0.55	0.06
9	3.0	0.4	2.43	4.97	0.64	0.15
10	5.0	0.1	0.36	1.88	0.42	0.13
11	5.0	0.2	0.33	1.84	0.48	0.04
12	5.0	0.4	0.46	1.98	0.43	0.02

CONCLUSIONS

Large number of cases analysed in this study have emphasized on the importance of correctly assigning the values of foundation parameters in the dynamic analysis of gravity dams. Foundation flexibility indicated by the impedance ratio, plays an important role in the response of a concrete gravity dam. The natural frequency of the dam foundation system increases with the increase in the impedance of the foundation. Out of the range of values used in the present study, the values of 0.25 and 5.0 for the impedance contrast are the extreme values with 0.25 being too flexible which cannot be expected in the hard rock foundation for gravity dams in practical reality and the value of 5.0 indicates a rigid foundation, which also cannot exist in actuality. The values for impedance contrast of 1.0 and 3.0 are the practical values and the responses of the dam obtained for these values are in the expected range in the present analysis. The response quantities are also sensitive to the magnitudes of Poisson's ratio of the foundation rock used in the analysis. In the present investigation, the range of values studied for Poisson's ratio is from 0.10 to 0.40. These are again extreme values and therefore, adoption of value around 0.20 is appropriate for the dynamic analysis which is as per the USBR recommendations. The dam section with unusual top width and excessive freeboard poses a problem as far as dynamic behaviour is concerned. Even though static analysis does not show any distress for such a case, the detailed FEM dynamic analysis indicates the detrimental effect of abnormal mass concentration especially at the top elevation of gravity dam cross section.

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